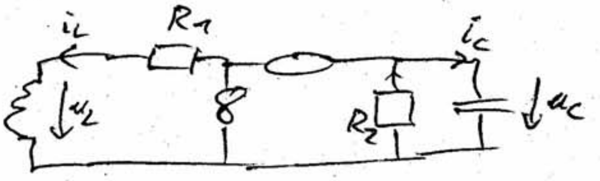
Aufgabe 7

a)



$$\left.\begin{array}{ll} i_C = C\dot{u}_C & i_C = -\frac{u_C}{R_2} \\ u_L = L\dot{i}_L & u_L = -R_1 i_L + u_C \end{array}\right\} \Rightarrow \begin{array}{l} \dot{i}_L = -\frac{R_1}{L} i_L + \frac{1}{L} u_C \\ \dot{u}_C = -\frac{u_C}{C R_2} \end{array} \qquad \begin{bmatrix} \dot{i}_L \\ \dot{u}_C \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L} & \frac{1}{L} \\ 0 & -\frac{1}{R_2 C} \end{bmatrix} \begin{bmatrix} i_L \\ u_C \end{bmatrix}$$

b) $$\det\begin{pmatrix} -\frac{R_1}{L} - \lambda & \frac{1}{L} \\ 0 & -\frac{1}{R_2 C} - \lambda \end{pmatrix} = \left(\lambda + \frac{R_1}{L}\right)\left(\lambda + \frac{1}{R_2 C}\right) \stackrel{!}{=} 0$$

$$\lambda_1 = -\frac{R_1}{L} \qquad \lambda_2 = -\frac{1}{R_2 C} \qquad \mathrm{Re}\{\lambda_i\} < 0$$

$\Rightarrow$ Schaltung für alle Bauelementwerte stabil

c) Eigenvektoren zur

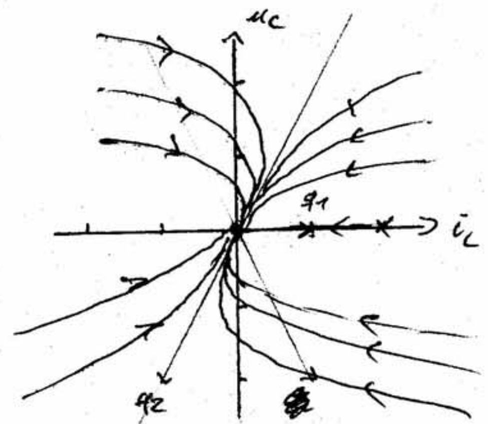
λ_1: $$\begin{bmatrix} 0 & \frac{1}{L} \\ 0 & -\frac{1}{R_2 C} + \frac{R_1}{L} \end{bmatrix} q_1 = \underline{0} \qquad q_1 = \begin{bmatrix} a \\ 0 \end{bmatrix}$$

λ_2: $$\begin{bmatrix} -\frac{R_1}{L} + \frac{1}{R_2 C} & \frac{1}{L} \\ 0 & 0 \end{bmatrix} q_2 = \underline{0} \qquad q_2 = \begin{bmatrix} -\frac{1}{L} \\ -\frac{R_1}{L} + \frac{1}{R_2 C} \end{bmatrix}$$

$$\underline{x} := \begin{bmatrix} i_L \\ u_C \end{bmatrix}$$

$$\underline{x}(t) = c_1 e^{-\frac{R_1}{L}t}\begin{pmatrix} a \\ 0 \end{pmatrix} + c_2 e^{-\frac{1}{R_2 C}t}\begin{pmatrix} -\frac{1}{L} \\ -\frac{R_1}{L} + \frac{1}{R_2 C} \end{pmatrix} = \begin{bmatrix} a e^{-\frac{R_1}{L}t} & -\frac{1}{L} e^{-\frac{1}{R_2 C}t} \\ 0 & \left(-\frac{R_1}{L} + \frac{1}{R_2 C}\right) e^{-\frac{1}{R_2 C}t} \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

d), f)



$$q_1 = \begin{bmatrix} 1A \\ 0 \end{bmatrix}$$

$$q_2 = \begin{bmatrix} -1A \\ -2V \end{bmatrix}$$

$$\underline{x}(t) = c_1 e^{-3s^{-1}t}\begin{pmatrix} 1A \\ 0 \end{pmatrix} + c_2 e^{-1s^{-1}t}\begin{pmatrix} -1A \\ -2V \end{pmatrix}$$

langsamer EV: $\begin{pmatrix} -1A \\ -2V \end{pmatrix}$

e) $$\underline{x}(0) = \begin{bmatrix} 1\frac{1}{s} & -1\frac{1}{s} \\ 0 & -2\frac{1}{s} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \stackrel{!}{=} \begin{pmatrix} 2A \\ 0V \end{pmatrix} \quad \Rightarrow \quad \begin{array}{l} c_1 = 2\,\mathrm{Vs} \\ c_2 = 0 \end{array}$$

$$\Rightarrow \underline{x}(t) = \begin{bmatrix} 2A e^{-3s^{-1}t} \\ 0 \end{bmatrix}$$

g)

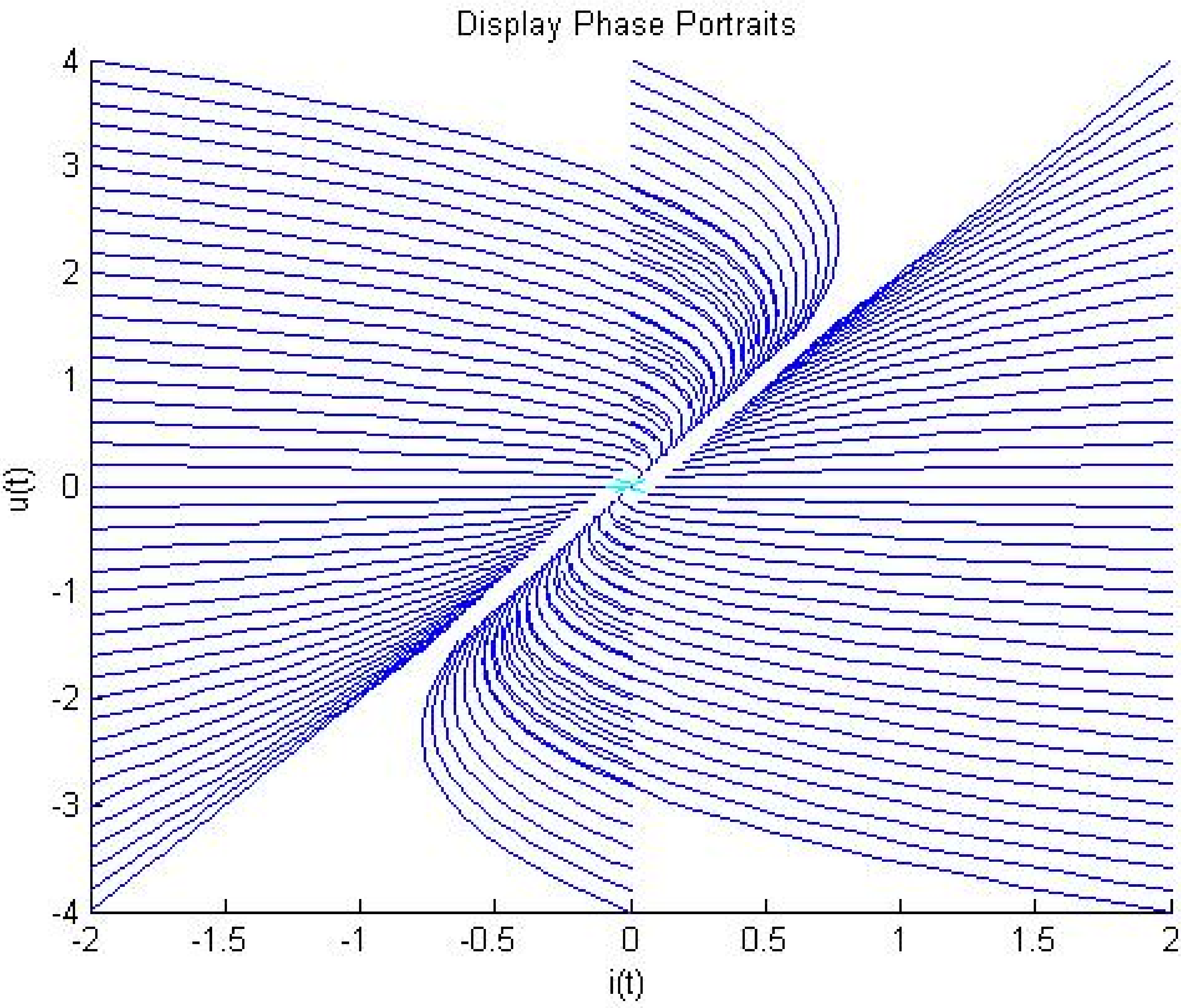
$$\det\begin{pmatrix} -\lambda & 1F^{-1} \\ -1H^{-1} & -\lambda \end{pmatrix} = \lambda^2 + 1s^{-2} = 0 \quad \Rightarrow \lambda = \pm i s^{-1} = \alpha + i\beta \qquad \alpha = 0;\ \beta = 1s^{-1}$$

$$\begin{pmatrix} -is^{-1} & 1F^{-1} \\ -1H^{-1} & -is^{-1} \end{pmatrix} q = \underline{0} \qquad \Rightarrow q = \begin{pmatrix} -1F^{-1} \\ -is^{-1} \end{pmatrix} = \underbrace{\begin{pmatrix} -1F^{-1} \\ 0 \end{pmatrix}}_{q_r} + i \underbrace{\begin{pmatrix} 0 \\ -1s^{-1} \end{pmatrix}}_{q_i}$$

allg. Lösung:

$$x(t) = c_1 e^{\alpha t}\left[\cos(\beta t) q_r - \sin(\beta t) q_i\right] + c_2 e^{\alpha t}\left[\sin(\beta t) q_r + \cos(\beta t) q_i\right] =$$

$$= c_1\left(\begin{bmatrix} -1F^{-1} \\ 0 \end{bmatrix} \cos(1s^{-1} t) - \begin{bmatrix} 0 \\ -s^{-1} \end{bmatrix} \sin(1s^{-1} t)\right) + c_2\left(\begin{bmatrix} -1F^{-1} \\ 0 \end{bmatrix} \sin(1s^{-1} t) + \begin{bmatrix} 0 \\ -1s^{-1} \end{bmatrix} \cos(1s^{-1} t)\right)$$


Display Phase Portraits
u(t)
4
3
2
1
0
-1
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-2
-1.5
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-0.5
0
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i(t)