Aufgabe 9

a) $u_x = Z\, i_x \quad \Rightarrow \quad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \underset{\sim}{R} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

Widerstandsmatrix

$u_1 = R i_2$

$u_2 = dR i_2 + aR i_1$

$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & R \\ aR & dR \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

b) $\begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{R}{L} \\ -\frac{aR}{L} & -\frac{dR}{L} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

c) $\det \begin{pmatrix} -\lambda & -\frac{R}{L} \\ -\frac{aR}{L} & -\frac{dR}{L} - \lambda \end{pmatrix} = \lambda \left(\lambda + \frac{dR}{L}\right) - a \frac{R^2}{L^2} =$

$= \lambda^2 + \frac{dR}{L} \lambda - a \frac{R^2}{L^2} \stackrel{!}{=} 0$

$\lambda_{1/2} = \frac{-\frac{dR}{L} \pm \sqrt{\frac{d^2 R^2}{L^2} + 4a \frac{R^2}{L^2}}}{2} = \frac{R}{L} \cdot \frac{-d \pm \sqrt{d^2 + 4a}}{2}$

stabil für $\mathrm{Re}\{\lambda_{1/2}\} < 0$

$\underline{d > 0} \qquad -d \pm \sqrt{d^2 + 4a} < 0$

$\underline{a < 0}$

d) $\begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{R}{L} \\ -\frac{R}{L} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

EW: $\det \begin{pmatrix} -\lambda & -\frac{R}{L} \\ -\frac{R}{L} & -\lambda \end{pmatrix} = \lambda^2 - \frac{R^2}{L^2} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad \lambda_{12} = \pm \frac{R}{L}$

EV zu $\lambda_1 = +\frac{R}{L} \qquad \begin{pmatrix} -\frac{R}{L} & -\frac{R}{L} \\ -\frac{R}{L} & -\frac{R}{L} \end{pmatrix} q_1 = \underline{0} \qquad q_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} A$

$\lambda_2 = -\frac{R}{L} \qquad \begin{pmatrix} \frac{R}{L} & -\frac{R}{L} \\ -\frac{R}{L} & \frac{R}{L} \end{pmatrix} q_2 = \underline{0} \qquad q_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} A$

$\underline{x} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad \underset{\sim}{Q} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \underset{\sim}{Q}^{-1} = -\frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$\underline{x} := \underset{\sim}{Q} \xi$ mit $\dot{\underline{x}} = \underset{\sim}{A} \underline{x}$ folgt:

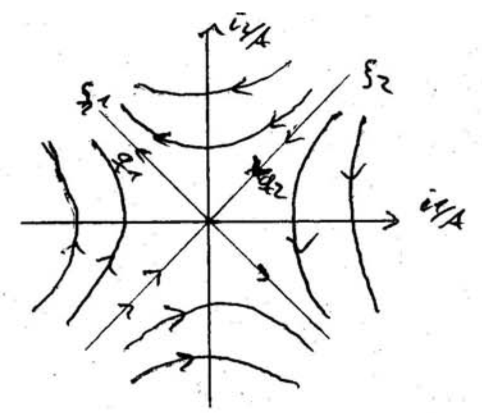
$\underset{\sim}{Q} \dot{\xi} = \underset{\sim}{A} \underset{\sim}{Q} \xi \quad \Rightarrow \quad \dot{\xi} = \underset{\sim}{Q}^{-1} \underset{\sim}{A} \underset{\sim}{Q} \xi = \begin{bmatrix} \frac{R}{L} & 0 \\ 0 & -\frac{R}{L} \end{bmatrix} \xi$

$\dot{\xi}_1 = \frac{R}{L} \xi_1 \quad \Rightarrow \quad \xi_1(t) = c_1 e^{\frac{R}{L} t}$

$\xi_2(t) = c_2 e^{-\frac{R}{L} t}$

$\underline{x} = \begin{bmatrix} \underbrace{-1}_{q_1} & \underbrace{1}_{q_2} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 e^{\frac{R}{L} t} \\ c_2 e^{-\frac{R}{L} t} \end{bmatrix} = c_1 e^{\frac{R}{L} t} q_1 + c_2 e^{-\frac{R}{L} t} q_2$

$\lambda_2 < 0 < \lambda_1 \quad \Rightarrow$ GGP ist Sattelpunkt



e) $\begin{bmatrix}\dot i_1\\ \dot i_2\end{bmatrix} = \begin{bmatrix}0 & -\frac{R}{L}\\ \frac{R}{L} & 0\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix}$

$$\det\begin{pmatrix}-\lambda & -\frac{R}{L}\\ \frac{R}{L} & -\lambda\end{pmatrix} = \lambda^2 + \frac{R^2}{L^2} \stackrel{!}{=} 0 \qquad \lambda = \pm i\frac{R}{L}$$

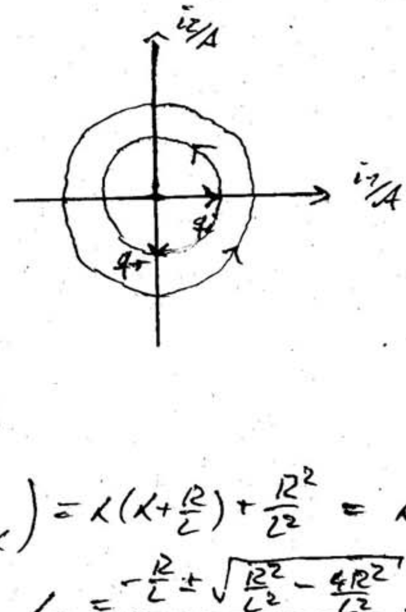
EW rein komplex $\Rightarrow$ GGP ist Wirbelpunkt

EV zu $\lambda = i\frac{R}{L}$

$$\begin{pmatrix}-i\frac{R}{L} & -\frac{R}{L}\\ \frac{R}{L} & -i\frac{R}{L}\end{pmatrix} q \stackrel{!}{=} \underline{0} \qquad q = \begin{bmatrix}i\\ 1\end{bmatrix}A = \begin{bmatrix}0\\ 1\end{bmatrix}A + i\begin{bmatrix}1\\ 0\end{bmatrix}A$$

$\underline{x} := \begin{bmatrix}i_1\\ i_2\end{bmatrix}$

$$\underline{x}(t) = c_1\left(\begin{bmatrix}0\\ 1A\end{bmatrix}\cos\left(\frac{R}{L}t\right) - \begin{bmatrix}1A\\ 0\end{bmatrix}\sin\left(\frac{R}{L}t\right)\right) + c_2\left(\begin{bmatrix}0\\ 1A\end{bmatrix}\sin\left(\frac{R}{L}t\right) + \begin{bmatrix}1A\\ 0\end{bmatrix}\cos\left(\frac{R}{L}t\right)\right)$$



f) $\begin{bmatrix}\dot i_1\\ \dot i_2\end{bmatrix} = \begin{bmatrix}0 & -\frac{R}{L}\\ \frac{R}{L} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix}$

$$\det\begin{pmatrix}-\lambda & -\frac{R}{L}\\ \frac{R}{L} & -\frac{R}{L}-\lambda\end{pmatrix} = \lambda\left(\lambda+\frac{R}{L}\right) + \frac{R^2}{L^2} = \lambda^2 + \frac{R}{L}\lambda + \frac{R^2}{L^2} \stackrel{!}{=} 0$$

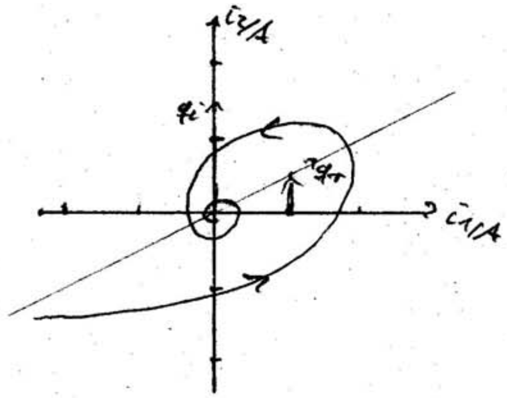
$$\lambda_{1/2} = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4R^2}{L^2}}}{2} = \frac{-\frac{R}{L} \pm i\frac{R}{L}\sqrt{3}}{2} = -\frac{R}{2L} \pm i\frac{R}{2L}\sqrt{3}$$

EV

$$\begin{pmatrix}\frac{R}{2L}(1+\sqrt{3}i) & -\frac{R}{L}\\ \frac{R}{L} & -\frac{R}{L}+\frac{R}{2L}(1+\sqrt{3}i)\end{pmatrix} q = \underline{0}$$

$$\begin{pmatrix}\frac{R}{2L}(1+\sqrt{3}i) & -\frac{R}{L}\\ \frac{R}{L} & \frac{R}{2L}(-1+\sqrt{3}i)\end{pmatrix} q = \underline{0} \qquad q = \begin{bmatrix}1\\ \frac{1}{2}+\frac{\sqrt{3}}{2}i\end{bmatrix}A \Rightarrow q_r = \begin{bmatrix}1\\ \frac{1}{2}\end{bmatrix}A \quad q_i = \begin{bmatrix}0\\ \frac{\sqrt{3}}{2}\end{bmatrix}A$$

$$\underline{x}(t) = c_1\exp\left(-\frac{R}{2L}t\right)\left(\begin{bmatrix}1\\ \frac{1}{2}\end{bmatrix}A\cos\left(\frac{\sqrt{3}R}{2L}t\right) - \begin{bmatrix}0\\ \frac{\sqrt{3}}{2}\end{bmatrix}A\sin\left(\frac{\sqrt{3}R}{2L}t\right)\right) + c_2\exp\left(-\frac{R}{2L}t\right)\left(\begin{bmatrix}1\\ \frac{1}{2}\end{bmatrix}A\sin\left(\frac{\sqrt{3}R}{2L}t\right) + \begin{bmatrix}0\\ \frac{\sqrt{3}}{2}\end{bmatrix}A\cos\left(\frac{\sqrt{3}R}{2L}t\right)\right)$$



Aufgabe 10

a) $\dot{u}_1 = \frac{1}{C} i_1 = \frac{1}{C}\left(+\frac{u_1}{R} + I' + i'\right) = \frac{1}{1F}\left(\frac{u_1}{1\Omega} + 2A + 1S\,u_2\right) := f_1(u_1, u_2)$

$\dot{u}_2 = \frac{1}{C} i_2 = \frac{1}{C}\left(-\frac{u_2}{R} + I'' + \left(\frac{-1S}{1V}\right)u_1^2\right) = \frac{1}{1F}\left(\frac{u_2}{1\Omega} + 4A - \frac{1S}{1V}u_1^2\right) := f_2(u_1, u_2)$

b) $\dot{u}_1 = 0 \quad \dot{u}_2 = 0$

$\frac{u_1}{1\Omega} + 2A + 1S\,u_2 = 0 \quad \Rightarrow u_2 = -\frac{u_1}{1\Omega \cdot 1S} - \frac{2A}{1S} = -u_1 - 2V$

$\frac{u_2}{1\Omega} + 4A - \frac{1S}{1V}u_1^2 = 0$

$\Rightarrow -\frac{u_1 + 2V}{1\Omega} + 4A - \frac{1A}{1V^2}u_1^2 = 0$

$-\frac{1A}{1V^2}u_1^2 - \frac{1A\,u_1}{1V} + 2A = 0 \quad \Leftrightarrow u_{1_{1/2}} = \frac{1 \pm \sqrt{1+8}}{-2}V = \frac{1 \pm 3}{-2}V$

$u_{1_1} = 1V \quad u_{1_2} = -2V$

$u_{2_1} = -3V \quad u_{2_2} = 0V$

$\underline{x}_{\infty 1} = \begin{pmatrix} 1V \\ -3V \end{pmatrix} \quad \underline{x}_{\infty 2} = \begin{pmatrix} -2V \\ 0V \end{pmatrix}$

c)

$J(\underline{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \end{bmatrix} = \begin{bmatrix} 1s^{-1} & 1s^{-1} \\ -\frac{2}{sV}u_1 & 1s^{-1} \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & \frac{1S}{C} \\ \frac{-2S\,u_1}{1V \cdot C} & -\frac{1}{RC} \end{bmatrix}$

d) $\Delta\dot{\underline{x}} = J\,\Delta\underline{x}$ Satz von Hartmann

pos Im-Teil
$\Rightarrow q_r$ nach $-q_i$ Drehe

$J\big|_{x_{\infty 1}} = \begin{pmatrix} 1s^{-1} & 1s^{-1} \\ -2s^{-1} & 1s^{-1} \end{pmatrix}$

$\det\begin{pmatrix} 1s^{-1} - \lambda & 1s^{-1} \\ -2s^{-1} & 1s^{-1} - \lambda \end{pmatrix} = (1s^{-1} - \lambda)^2 + 2s^{-2} \stackrel{!}{=} 0 \quad \Leftrightarrow \lambda = 1s^{-1} \pm j\sqrt{2}\,s^{-1}$

$\Rightarrow$ instabiler Strudel

EV zu $1s^{-1} + j\sqrt{2}\,s^{-1}$: $\begin{pmatrix} -j\sqrt{2}\,s^{-1} & 1s^{-1} \\ -2s^{-1} & -j\sqrt{2}\,s^{-1} \end{pmatrix} q = \underline{0} \Rightarrow q = \begin{bmatrix} -1V \\ -j\sqrt{2}\,V \end{bmatrix} \quad q_r = \begin{bmatrix} 1V \\ 0V \end{bmatrix} \quad q_i = \begin{bmatrix} 0V \\ \sqrt{2}\,V \end{bmatrix}$

e) $J\Big|_{x_{02}} = \begin{pmatrix} 1s^{-1} & 1s^{-1} \\ 4s^{-1} & 1s^{-1} \end{pmatrix}$

$$\det \begin{pmatrix} 1s^{-1}-\lambda & 1s^{-1} \\ 4s^{-1} & 1s^{-1}-\lambda \end{pmatrix} = (1s^{-1}-\lambda)^2 - 4s^{-2} \overset{!}{=} 0 \iff \lambda = 1s^{-1} \pm 2s^{-1}$$

$$\lambda_1 = -1s^{-1} \quad \lambda_2 = 3s^{-1}$$

$$\lambda_1 < 0 < \lambda_2 \implies \text{Sattelpunkt}$$

EV zu λ_1: $\begin{pmatrix} 2s^{-1} & 1s^{-1} \\ 4s^{-1} & 2s^{-1} \end{pmatrix} q_1 = \underline{0} \iff q_1 = \begin{bmatrix} 1V \\ -2V \end{bmatrix}$

λ_2: $\begin{pmatrix} -2s^{-1} & 1s^{-1} \\ 4s^{-1} & -2s^{-1} \end{pmatrix} q_2 = \underline{0} \iff q_2 = \begin{bmatrix} 1V \\ 2V \end{bmatrix}$

f)