Aufgabe 11

a) Spannungsteiler:

$$u_2 = \frac{j\omega L}{\frac{1}{j\omega C} + j\omega L} u_1 = \frac{(j\omega)^2 LC}{1+(j\omega)^2 LC} u_1 = \frac{1}{1+\frac{1}{(j\omega)^2 LC}} u_1$$

b) $\left.\begin{array}{l} \omega = 0 \Rightarrow u_2 = 0 \\ \omega \to \infty \Rightarrow u_2 \to u_1 \end{array}\right\} \Rightarrow$ Hochpassverhalten

c) Spannungsteiler:

$$u_2 = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L} u_1 = \frac{1}{1+(j\omega)^2 LC} u_1$$

$$\Rightarrow H(p) = \frac{u_2}{u_1} = \frac{1}{1+p^2 LC}$$

d) keine Nullstellen

Polstellen: $1 + p^2 LC = 0$

$$p^2 = -\frac{1}{LC}$$

$$j\omega = \pm \frac{j}{\sqrt{LC}} \Leftrightarrow \omega = \pm \frac{1}{\sqrt{LC}}$$

e) $H(j\omega) = \frac{1}{1+(j\omega)^2 LC} = \frac{1}{1-\omega^2 LC} e^{j0}$

$\cancel{f)\ V(\omega) = \frac{1}{1-\omega^2 LC}}$

$\cancel{\frac{dV}{d\omega} = \frac{+2\omega LC}{(1-\omega^2 LC)} \overset{!}{=} 0 \Leftrightarrow \omega = 0}$

f) $u_2 = \frac{R}{2R + \frac{1}{j\omega C}} u_1 = \frac{j\omega RC}{2j\omega RC + 1} u_1$

$$H(j\omega) = \frac{u_2}{u_1} = \frac{j\omega RC}{1+2j\omega RC} = \frac{j\omega RC(1-2j\omega RC)}{1+4(\omega RC)^2} = \frac{2\omega^2R^2C^2 + j\omega RC}{1+(2\omega RC)^2} = \frac{\omega RC(2\omega RC + j)}{1+(2\omega RC)^2} =$$

$$= \frac{\omega RC\sqrt{1+(2\omega RC)^2}}{1+(2\omega RC)^2} e^{j\arctan\frac{1}{2\omega RC}} = \frac{\omega RC}{\sqrt{1+(2\omega RC)^2}} e^{j\cdot\arctan\frac{1}{2\omega RC}}$$

g) $V(\omega) = \frac{\omega RC}{\sqrt{1+(2\omega RC)^2}}$ $\quad \frac{dV}{d\omega} = \frac{RC\sqrt{1+(2\omega RC)^2} - \omega RC\frac{8\omega R^2C^2}{2\sqrt{1+(2\omega RC)^2}}}{1+(2\omega RC)^2} =$

$$= \frac{RC}{\sqrt{1+(2\omega RC)^2}} - \frac{4\omega^2 R^3 C^3}{(1+(2\omega RC)^2)^{3/2}} = \frac{RC}{(1+4\omega^2R^2C^2)^{3/2}} \overset{!}{=} 0$$

$\Rightarrow \omega \to \infty$

$V(\omega_{max}) = \frac{1}{\sqrt{\frac{1}{\omega^2R^2C^2} + 4}}$ mit $\omega \to \infty \Rightarrow V_{max} = \frac{1}{2}$

h) $V(\omega_g) = \frac{\omega_g RC}{\sqrt{1+(2\omega_g RC)^2}} \overset{!}{=} \frac{1}{2\sqrt{2}} \Rightarrow \omega_g^2 R^2 C^2 = \frac{1+4\omega_g^2R^2C^2}{8}$

$$\frac{1}{2}\omega_g^2 R^2 C^2 = \frac{1}{8} \Rightarrow \omega_g = \frac{1}{2RC}$$

i) $\varphi(0) = \arctan(\text{„}\infty\text{"}) = \frac{\pi}{2}$

$\varphi(\infty) = \arctan(0) = 0$

$\varphi(\omega_g) = \arctan(1) = \frac{\pi}{4}$