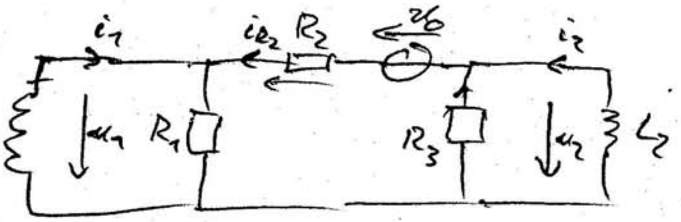
Aufgabe 8.

a) $u_1 = -L_1 \dot{i}_1$, $u_2 = -L_2 \dot{i}_2$ $\qquad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \underline{R} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$ $\Rightarrow$ Widerstandsbeschreibung



(I) $\quad i_{R_2} + i_1 = \frac{u_1}{R_1}$

(II) $\quad -\frac{u_2}{R_3} + i_2 = i_{R_2}$

(III) $\quad i_{R_2} = \frac{-u_0 + u_2 - u_1}{R_2}$

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aus (III): $\quad u_2 = R_2 i_{R_2} + u_0 + u_1 \qquad (*)$

in (II): $\quad -\frac{R_2}{R_3} i_{R_2} - \frac{u_0}{R_3} - \frac{u_1}{R_3} + i_2 = i_{R_2} \Rightarrow i_{R_2}\left(1 + \frac{R_2}{R_3}\right) = -\frac{u_0}{R_3} - \frac{u_1}{R_3} + i_2$

$$\Rightarrow i_{R_2} = \frac{-u_0 - u_1 + R_3 i_2}{R_2 + R_3}$$

in (I): $\quad -\frac{u_0}{R_2+R_3} + \frac{R_3}{R_2+R_3} i_2 + i_1 = u_1\left(\frac{1}{R_1} + \frac{1}{R_2+R_3}\right)$

$$u_1 = \frac{-R_1 u_0 + R_1 R_3 i_2 + R_1(R_2+R_3) i_1}{R_1 + R_2 + R_3}$$

mit (*): $\quad u_2 = \frac{-R_2}{R_2+R_3} u_0 - \frac{R_2}{R_2+R_3} u_1 + \frac{R_2 R_3}{R_2+R_3} i_2 + u_0 + u_1 =$

$$= \frac{-R_2}{R_2+R_3} u_0 + \frac{R_2 R_3}{R_2+R_3} i_2 + u_0 + u_1\left(1 - \frac{R_2}{R_2+R_3}\right) =$$

$$= u_0\left(1 - \frac{R_2}{R_2+R_3}\right) + \frac{R_2 R_3}{R_2+R_3} i_2 + \frac{R_3}{R_2+R_3}\left(-\frac{R_1}{R_1+R_2+R_3} u_0 + \frac{R_1 R_3 i_2}{R_1+R_2+R_3} + \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} i_1\right) =$$

$$= u_0\left(\frac{R_3}{R_2+R_3} - \frac{R_1 R_3}{(R_1+R_2+R_3)(R_2+R_3)}\right) + \frac{R_1 R_3}{R_1+R_2+R_3} i_1 + \left(\frac{R_2 R_3}{R_2+R_3} + \frac{R_1 R_3^2}{(R_2+R_3)(R_1+R_2+R_3)}\right) i_2 =$$

$$= \frac{\cancel{R_1 R_3} + R_2 R_3 + R_3^2 \cancel{- R_1 R_3}}{(R_2+R_3)(R_1+R_2+R_3)} u_0 + \frac{R_1 R_3}{R_1+R_2+R_3} i_1 + \frac{R_1 R_2 R_3 + R_2^2 R_3 + R_2 R_3^2 + R_1 R_3^2}{(R_2+R_3)(R_1+R_2+R_3)} i_2$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} & \frac{R_1 R_3}{R_1+R_2+R_3} \\ \frac{R_1 R_3}{R_1+R_2+R_3} & \frac{R_1 R_2 R_3 + R_2^2 R_3 + R_2 R_3^2 + R_1 R_3^2}{(R_2+R_3)(R_1+R_2+R_3)} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} \frac{-R_1}{R_1+R_2+R_3} \\ \frac{R_2 R_3 + R_3^2}{(R_2+R_3)(R_1+R_2+R_3)} \end{bmatrix} u_0$$

b) $-L_1\dot i_1 = u_1$

$-L_2\dot i_2 = u_2$

$$\begin{bmatrix}\dot i_1\\ \dot i_2\end{bmatrix} = \begin{bmatrix} -\frac{R_1(R_2+R_3)}{L_1(R_1+R_2+R_3)} & -\frac{R_1R_3}{L_1(R_1+R_2+R_3)} \\ -\frac{R_1R_3}{L_2(R_1+R_2+R_3)} & -\frac{R_1R_2R_3+R_2^2R_3+R_2R_3^2+R_1R_3^2}{L_2(R_2+R_3)(R_1+R_2+R_3)}\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix} + \begin{bmatrix}\frac{+R_1}{L_1(R_1+R_2+R_3)} \\ -\frac{R_2R_3+R_3^2}{L_2(R_2+R_3)(R_1+R_2+R_3)}\end{bmatrix} u_0$$

$$\begin{bmatrix}\dot i_1\\ \dot i_2\end{bmatrix} = \begin{bmatrix} -\frac{R_1(R_2+R_3)}{L_1(R_1+R_2+R_3)} & -\frac{R_1R_3}{L_1(R_1+R_2+R_3)} \\ -\frac{R_1R_3}{L_2(R_1+R_2+R_3)} & -\frac{R_3(R_1+R_2)}{L_2(R_1+R_2+R_3)}\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix} + \begin{bmatrix}\frac{+R_1}{L_1(R_1+R_2+R_3)} \\ \frac{-R_3}{L_2(R_1+R_2+R_3)}\end{bmatrix} u_0$$

c) $$\begin{bmatrix}\dot i_1\\ \dot i_2\end{bmatrix} = \begin{bmatrix}-3s^{-1} & -2s^{-1}\\ -1s^{-1} & -2s^{-1}\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix} + \begin{bmatrix}+4\frac{A}{s}\\ -4\frac{A}{s}\end{bmatrix}$$

d) $\underline{\dot x} = \underline{A}\,\underline{x} + \underline{b}v$

mit $\underline{x}' = \underline{x} + \underline{A}^{-1}\underline{b}v$ folgt $\underline{x} = \underline{x}' - \underline{A}^{-1}\underline{b}v$

eingesetzt:

$$\underline{\dot x}' = \underline{A}\,\underline{x}' - \underline{A}\,\underline{A}^{-1}\underline{b}v + \underline{b}v = \underline{A}\,\underline{x}'$$

$$\underline{A}^{-1} = \frac{1}{\det A}\begin{bmatrix}a_{22} & -a_{12}\\ -a_{21} & a_{11}\end{bmatrix} = \frac{1}{4s^{-2}}\begin{bmatrix}-2s^{-1} & +2s^{-1}\\ +1s^{-1} & -3s^{-1}\end{bmatrix}$$

$$\underline{x} = \underline{x}' - \frac{1}{4}\begin{bmatrix}-2s & +2s\\ +1s & -3s\end{bmatrix}\cdot\begin{bmatrix}+4\frac{A}{s}\\ -4\frac{A}{s}\end{bmatrix} = \underline{x}' - \begin{bmatrix}-4A\\ 4A\end{bmatrix}$$

e) EW:
$$\det\begin{pmatrix}-3s^{-1}-\lambda & -2s^{-1}\\ -1s^{-1} & -2s^{-1}-\lambda\end{pmatrix} = (\lambda+3s^{-1})(\lambda+2s^{-1}) - 2s^{-2} =$$
$$= \lambda^2 + 5s^{-1}\lambda + 4s^{-2} \overset{!}{=} 0$$
$$\lambda_{1,2} = \frac{-5s^{-1} \pm \sqrt{25s^{-2} - 16s^{-2}}}{2} = \frac{-5s^{-1} \pm 3s^{-1}}{2}$$
$\lambda_1 = -4s^{-1}$ $\quad\lambda_2 = -1s^{-1}$ $\quad\Rightarrow$ Schaltung stabil

EV zu

λ_1: $\begin{pmatrix}+1s^{-1} & -2s^{-1}\\ -1s^{-1} & +2s^{-1}\end{pmatrix}\underline{q}_1 = \underline{0}$ $\qquad \underline{q}_1 = \begin{bmatrix}2A\\ +1A\end{bmatrix}$

λ_2: $\begin{pmatrix}-2s^{-1} & -2s^{-1}\\ -1s^{-1} & -1s^{-1}\end{pmatrix}\underline{q}_2 = \underline{0}$ $\qquad \underline{q}_2 = \begin{bmatrix}1A\\ -1A\end{bmatrix}$

Lösung:

$$\underline{x}' = c_1 e^{-4s^{-1}t}\cdot\begin{bmatrix}2A\\ 1A\end{bmatrix} + c_2 e^{-1s^{-1}t}\cdot\begin{bmatrix}1A\\ -1A\end{bmatrix}$$

$$\Rightarrow \underline{x} = c_1 e^{-4s^{-1}t}\cdot\begin{bmatrix}2A\\ 1A\end{bmatrix} + c_2 e^{-1s^{-1}t}\cdot\begin{bmatrix}1A\\ -1A\end{bmatrix} + \begin{bmatrix}4A\\ -4A\end{bmatrix}$$

f)